CCGPS • 6th Grade Math Content Standards Unpacked

This document is an instructional support tool. It is adapted from documents created by the Partnership for Assessment of Readiness for College and Careers (PARCC), the Ohio Department of Education, the Arizona Department of Education, and the North Carolina Department of Public Instruction for the Common Core State Standards in Mathematics.

Highlighted standards are transition standards for Georgia's implementation of CCGPS in 2012-2013. The highlighted standards are included in the curriculum for two grade levels during the initial year of CCGPS implementation to ensure that students do not have gaps in their knowledge base. In 2013-2014 and subsequent years, the highlighted standards will not be taught at this grade level because students will already have addressed these standards the previous year.

What is the purpose of this document? To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

What is in the document? Descriptions of what each standard means a student will know, understand, and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send feedback? The explanations and examples in this document are intended to be helpful and specific. As this document is used, however, teachers and educators will find ways in which the unpacking can be improved and made more useful. Please send feedback to ljanes@bibb.k12.ga.us. Your input will be used to refine the unpacking of the standards.

Just want the standards alone? You can find the standards alone at www.georgiastandards.org.

Grade 6

Grade 6 Overview

Ratios and Proportional Relationships (RP)
- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System (NS)
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations (EE)
- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry (G)
- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (SP)
- Develop understanding of statistical variability.
- Summarize and describe distributions.

Key
Major Cluster
Supporting Cluster
Additional Cluster

Mathematical Practices (MP)
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Critical Areas
1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.
2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers.
3. Writing, interpreting, and using expressions and equations.
4. Developing understanding of statistical thinking.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Solve real-world and mathematical problems involving area, surface area and volume: In this cluster, students work on problems with areas of triangles and volumes of right rectangular prisms, which connects to work in the Expressions and Equations domain. In addition, another standard within this cluster asks students to draw polygons in the coordinate plane, which supports other work with the coordinate plane in The Number System domain.

Key Advances from Grade 5 to Grade 6	Fluency Expectations/Culminating Standards
• Students' prior understanding of and skill with multiplication, division and fractions contribute to their study of ratios, proportional relationships and unit rates (6.RP). • Students begin using properties of operations systematically to work with variables, variable expressions and equations (6.EE). • Students extend their work with the system of rational numbers to include using positive and negative numbers to describe quantities (6.NS.5), extending the number line and coordinate plane to represent rational numbers and ordered pairs (6.NS.6), and understanding ordering and absolute value of rational numbers (6.NS.7). • Having worked with measurement data in previous grades, students begin to develop notions of statistical variability, summarizing and describing distributions (6.SP).	• **6.NS.2** Students fluently divide multidigit numbers using the standard algorithm. This is the culminating standard for several years' worth of work with division of whole numbers. • **6.NS.3** Students fluently add, subtract, multiply and divide multidigit decimals using the standard algorithm for each operation. This is the culminating standard for several years' worth of work relating to the domains of Number and Operations in Base Ten, Operations and Algebraic Thinking, and Number and Operations — Fractions. • **6.NS.1** Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. This completes the extension of operations to fractions.
Examples of Major Within-Grade Dependencies	**Examples of Opportunities for In-Depth Focus**
• Equations of the form $px = q$ (6.EE.7) are unknown-factor problems; the solution will sometimes be the quotient of a fraction by a fraction (6.NS.1). • Solving problems by writing and solving equations (6.EE.7) involves not only an appreciation of how variables are used (6.EE.6) and what it means to solve an equation (6.EE.5) but also some ability to write, read and evaluate expressions in which letters stand for numbers (6.EE.2). • Students must be able to place rational numbers on a number line (6.NS.7) before they can place ordered pairs of rational numbers on a coordinate plane (6.NS.8). The former standard about ordering rational numbers is much more fundamental.	• **6.RP.3** When students work toward meeting this standard, they use a range of reasoning and representations to analyze proportional relationships. • **6.NS.1** This is a culminating standard for extending multiplication and division to fractions. • **6.NS.8** When students work with rational numbers in the coordinate plane to solve problems, they combine and consolidate elements from the other standards in this cluster. • **6.EE.3** By applying properties of operations to generate equivalent expressions, students use properties of operations that they are familiar with from previous grades' work with numbers — generalizing arithmetic in the process. • **6.EE.7** When students write equations of the form $x + p = q$ and $px = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems.[1]

[1] For example, supposed Daniel went to visit his grandmother, who gave him $5.50. Then he bought a book costing $9.20 and had $2.30 left. To find how much money he had before visiting his grandmother, an algebraic approach leads to the equation x + 5.50 – 9.20 = 2.30. An arithmetic approach without using variables at all would be to begin with 2.30, then add 9.20, then subtract 5.50. This yields the desired answer, but students will eventually encounter problems in which arithmetic approaches are unrealistically difficult and algebraic approaches must be used.

Examples of Opportunities for Connections among Standards, Clusters, or Domains	Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices
• Students' work with ratios and proportional relationships (6.RP) can be combined with their work in representing quantitative relationships between dependent and independent variables (6.EE.9). • Plotting rational numbers in the coordinate plane (6.NS.8) is part of analyzing proportional relationships (6.RP.3a, 7.RP.2) and will become important for studying linear equations (8.EE.8) and graphs of functions (8.F).[2] • Students use their skill in recognizing common factors (6.NS.4) to rewrite expressions (6.EE.3). • Writing, reading, evaluating and transforming variable expressions (6.EE.1–4) and solving equations and inequalities (6.EE.7–8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (6.G.2). • Working with data sets can connect to estimation and mental computation. For example, in a situation where there are 20 different numbers that are all between 8 and 10, one might quickly estimate the sum of the numbers as 9 × 20 = 180.	Mathematical practices should be evident *throughout* mathematics instruction and connected to all of the content areas highlighted above, as well as all other content areas addressed at this grade level. Mathematical tasks (short, long, scaffolded and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the practices follow. • Reading and transforming expressions involves seeing and making use of structure (MP.7). Relating expressions to situations requires making sense of problems (MP.1) and reasoning abstractly and quantitatively (MP.2). • The sequence of steps in the solution of an equation is a logical argument that students can construct and critique (MP.3). Such arguments require looking for and making use of structure (MP.7) and, over time, expressing regularity in repeated reasoning (MP.8). • Thinking about the point (1, *r*) in a graph of a proportional relationship with unit rate *r* involves reasoning abstractly and quantitatively (MP.2). The graph models with mathematics (MP.4) and uses appropriate tools strategically (MP.5). • Area, surface area and volume present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6). • Students think with precision (MP.6) and reason quantitatively (MP.2) when they use variables to represent numbers and write expressions and equations to solve a problem (6.EE.6–7). • Working with data gives students an opportunity to use appropriate tools strategically (MP.5). For example, spreadsheets can be powerful for working with a data set with dozens or hundreds of data points.

[2] While not required by the standards, it might be considered valuable to expose students to time series data and to time graphs as an appealing way to work with rational numbers in the coordinate plane (6.NS.8). For example, students could create time graphs of temperature measured each hour over a 24-hour period in a place where, to ensure a strong connection to rational numbers, temperature values might cross positive to negative during the night and back to positive the next day.

Standards for Mathematical Practices

The Common Core Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K – 12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematical Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Standards for Mathematical Content

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Document Contents for Grade 6

Contents Continued

Instructional Strategies

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and nonproportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percents of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percents are to be taught as a special type of rate. Provide students with opportunities to find percents in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	$4.96

Store B	
6 cans	3 cans
$4.50	$2.25

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, t-charts or double number line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.

Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$$\frac{2}{5} = \frac{6}{x}$$

Recognize that the relationship between 2 and 6 is 3 times; $2 \cdot 3 = 6$.

To find x, the relationship between 5 and x must also be 3 times. $3 \cdot 5 = x$; therefore, $x = 15$.

$$\frac{2}{5} = \frac{6}{15}$$

The final proportion.

Other ways to illustrate ratios that will help students see the relationships follow. Begin written representation of ratios with the words “out of” or “to” before using the symbolic notation of the colon and then the fraction bar; for example, 3 out of 7, 3 to 5, 6:7 and then 4/5.

Use skip counting as a technique to determine if ratios are equal.

Labeling units helps students organize the quantities when writing proportions.

$$\frac{3 \text{ eggs}}{2 \text{ cups of flour}} = \frac{z \text{ eggs}}{8 \text{ cups of flour}}$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

Instructional Resources/Tools

- 100 grids (10 × 10) for modeling percents
- Ratio tables to use for proportional reasoning
- Bar models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then expand the number of bars to show other equivalent ratios
- **Something Fishy**
  http://www.pbs.org/teachers/mathline/lessonplans/msmp/somethingfishy/somethingfishy_procedure.shtm
  Students will estimate the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure.
- **How Many Noses Are in Your Arm?**
  http://www.pbs.org/teachers/mathline/lessonplans/msmp/noses/noses_procedure.shtm
  Students will apply the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.
- **If You Hopped Like a Frog**
  http://ohiorc.org/for/math/bookshelf/detail.aspx?id=30&gid=2
  This book introduces the concepts of ratio and proportion by comparing what humans would be able to do if they had the capabilities of different animals.

Common Misconceptions

Fractions and ratios may represent different comparisons. Fractions always express a part-to-whole comparison, but ratios can express a part-to-whole comparison or a part-to-part comparison.

Even though ratios and fractions express a part-to-whole comparison, the addition of ratios and the addition of fractions are distinctly different procedures. When adding ratios, the parts are added, the wholes are added and then the total part is compared to the total whole. For example, (2 out of 3 parts) + (4 out of 5 parts) is equal to six parts out of 8 total parts (6 out of 8) if the parts are equal. When dealing with fractions, the procedure for addition is based on a common denominator:

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} \text{ which is equal to } \frac{22}{15}.$$

Often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

Connections – Critical Areas of Focus

This cluster is connected to the first Critical Area of Focus for 6th grade:

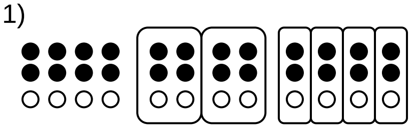
Through learning in **Ratios and Proportional relationships** students

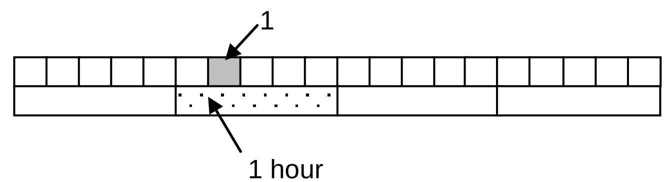
- Use reasoning about multiplication and division to solve ratio and rate problems about quantities.

Connections to Other Grade Levels

In 6th grade, students develop the foundational understanding of ratio and proportion that will be extended in 7th grade to include scale drawings, slope and real-world percent problems.

- Connect their understanding of multiplication and division with ratios and rates by viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities.
- Expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions.
- Solve a wide variety of problems involving ratios and rates.

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **DOK 1**	**6.MP.2.** Reason abstractly and quantitatively. **6.MP.6.** Attend to precision.	ratio ratio language relationship quantities	understand use describe	fractions	A ratio is a comparison of two quantities which can be written as *a* to *b*, $\frac{a}{b}$, or *a:b*. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2) and 2 black circles to 1 white circle (2:1)  Students should be able to identify all these ratios and describe them using "For every…., there are …"
6.RP.2. Understand the concept of a unit rate a/b associated with a ratio *a:b* with $b \neq 0$, and use rate	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.6**. Attend	unit rate a/b ratio a:b rate language	understand use	division of whole numbers and decimals	When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.

language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $^{3}/_{4}$ *cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Expectations for unit rates in this grade are limited to non-complex fractions.) **DOK 2**	to precision.	ratio relationship			A unit rate expresses a ratio as part-to-one. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates, however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule because at this level, students should primarily use reasoning to find these unit rates. **Examples:** • On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)? Solution: You can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\text{ hr}}{1\text{ mi}}$. Students can represent the relationship between 20 miles and 4 hours.  A simple modeling clay recipe calls for 1 cup corn starch, 2 cups salt, and 2 cups boiling water. How many cups of corn starch are needed to mix with each cup of salt?
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model	table equivalent ratios quantities whole number measurements values pairs coordinate plane ratios	make find plot use compare	make a table multiplication division write ratios simplify ratios plot points on a coordinate system compare fractions	Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard. **Example:**

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
DOK 1

with mathematics

6.MP.5. Use appropriate tools strategically.

6.MP.7. Look for and make use of structure.

At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54? To find the price of 1 book, divide $18 by 3. One book is $6. To find the price of 7 books, multiply $6 (the cost of one book) times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally and vertically. (Red numbers indicate solutions.)

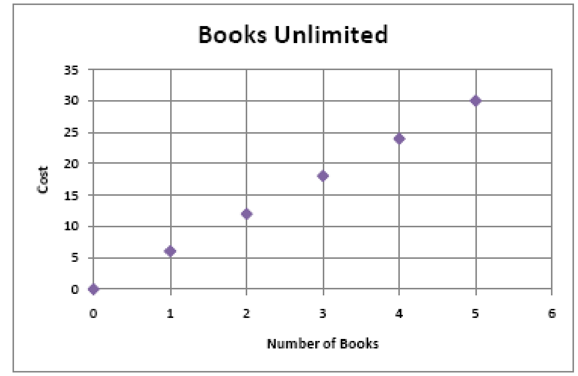
Number of Books	**Cost**
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain how you determined your answer.

Number of Books	**Cost**
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$.

The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane. Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:

<table>
<tr><td></td><td></td><td></td><td></td><td></td><td>
</td></tr>
<tr><td>b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
DOK 2</td><td>6.MP.1. Make sense of problems and persevere in solving them.

6.MP.2. Reason abstractly and quantitatively.

6.MP.4. Model with mathematics

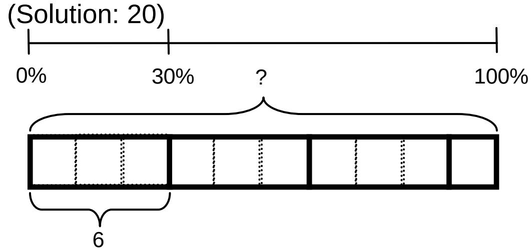
6.MP.5. Use appropriate tools strategically.

6.MP.7. Look for and make use of structure.</td><td>unit rate
unit price
constant speed</td><td>solve</td><td>write proportions
solve proportions
multiply fractions</td><td>Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

The ratio tables above use unit rate by determining the cost of one book. However, ratio tables can be used to solve problems with the use of a unit rate. For example, in trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

<table><tr><th>Peanuts</th><th>Chocolate</th></tr><tr><td></td><td></td></tr><tr><td>3</td><td>2</td></tr><tr><td></td><td></td></tr></table>
One possible way to solve this problem is to recognize that 3 cups of peanuts times 3 will give 9 cups. The amount of chocolate will also increase at the same rate (3 times) to give 6 cups of chocolate.

Students could also find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by 9 $9 \cdot \frac{2}{3}$, giving 6 cups of chocolate.</td></tr>
</table>

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics **6.MP.5**. Use appropriate tools strategically. **6.MP.7**. Look for and make use of structure.	percent quantity rate per 100 whole part	find solve	change percent to decimals change percent to fractions multiply fractions multiply decimals	This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents. Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent). For example, to find 40% of 30, students could use a 10 x 10 grid to represent the whole (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12. Students also find the whole, given a part and the percent. For example is 25% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream? Students can reason that if 25% is 6 and 100% is 4 times the 25%, then 6 times 4 would give 24 students in Mrs. Rutherford's class. Example: • If 6 is 30% of a value, what is that value? (Solution: 20) 
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics	ratio reasoning measurement units quantities	use convert manipulate transform multiply divide	write ratios conversion factors multiply and divide fractions	A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the quantity described in the numerator and denominator is the same. For example, $\frac{12\ inches}{1\ foot}$ is a conversion factor since the numerator and denominator name the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can be expressed as $\frac{1\ foot}{12\ inches}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel."

	6.MP.5. Use appropriate tools strategically. **6.MP.7**. Look for and make use of structure.				Students use ratios as conversion factors and the identity property for multiplication to convert ratio units. For example, how many centimeters are in 7 feet, given that 1 inch = 2.54 cm. $7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \cancel{\text{feet}} \times \frac{12 \cancel{\text{inches}}}{1 \cancel{\text{foot}}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{inch}}} =$ 7 x 12 x 2.54 cm = 213.36 cm **Note:** Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

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CCGPS Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

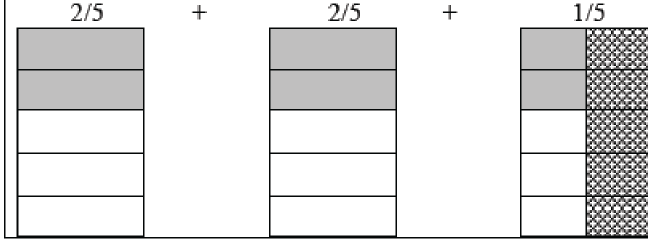
CCGPS	What does this standard mean that a student will know and be able to do?
CC.5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example:

There are $2\frac{1}{2}$ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. Two-fifths of the students on each bus are girls. How many busses would it take to carry ***only*** the girls?

Student 1
I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving 2 ½ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, 2/5 of the 1st and 2nd bus were both shaded, and 1/5 of the last bus was shaded.

2/5 + 2/5 + 1/5 = 5/5 = 1 whole bus.

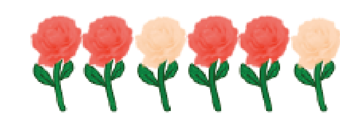


Student 2
2 ½ x 2/5=
I split the 2 ½ into 2 and ½
2 x 2/5 = 4/5
½ x 2/5 = 2/10
I then added 4/5 and 2/10. That equals 1 whole bus load.

Example:

Evan bought 6 roses for his mother. Two-thirds of them were red. How many red roses were there?

Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.



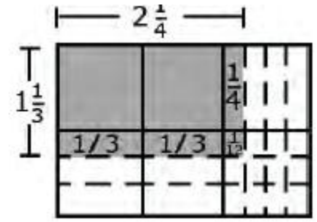
A student can use an equation to solve.

$$\frac{2}{3}\times 6 = \frac{12}{3} = 4 \text{ red roses}$$

Example:

Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ by $2\frac{1}{4}$ ft. What will be the area of the school flag?

A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by instead $1\frac{1}{3}$ of $2\frac{1}{4}$.



The explanation may include the following:

- First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.
- When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.
- Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
- $\frac{1}{3}$ times 2 is $\frac{2}{3}$.
- $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
- So the answer is $2\frac{1}{4}+\frac{2}{3}+\frac{1}{12}$ or $2\frac{3}{12}+\frac{8}{12}+\frac{1}{12}=2\frac{12}{12}=3$.

The Number System 6.NS

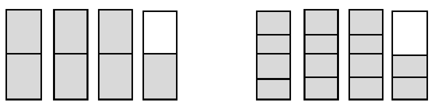
CCGPS Cluster: Apply and extend previous understands of multiplication and division to divide fractions by fractions.

Instructional Strategies

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example: 12÷3 means "How many groups of three would make 12?" Or, "How many in each of 3 groups would make 12?" Thus $^{7}/_{2} \div {}^{1}/_{4}$ can be solved the same way. "How many groups of $^{1}/_{4}$ make $^{7}/_{2}$?" Or,"How many objects in a group when $^{7}/_{2}$ fills $^{1}/_{4}$?"

Creating the picture that represents this problem makes seeing and proving the solutions easier.



Set the problem in context and represent the problem with a concrete or pictorial model.

$^{5}/_{4} \div {}^{1}/_{2}$: $^{5}/_{4}$ cups of nuts fills $^{1}/_{2}$ of a container. How many cups of nuts will fill the entire container?

Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

Instructional Resources/Tools

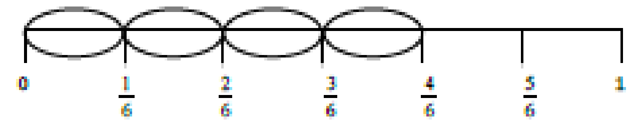
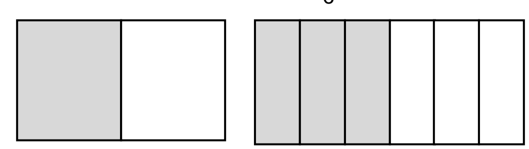
- **Models for Multiplying and Dividing Fractions**
  http://www.learner.org/courses/learningmath/number/session9/part_a/translating.html
  This teacher resource gives shows how the area model can be used in multiplication and division of fractions. There is also a section on the relationship to decimals.
- **Fractions – Rectangle Multiplication** (from the National Library of Virtual Manipulatives)
  http://nlvm.usu.edu/en/nav/frames_asid_194_g_3_t_1.html
  Use this virtual manipulative to graphically demonstrate, explore, and practice multiplying fractions.

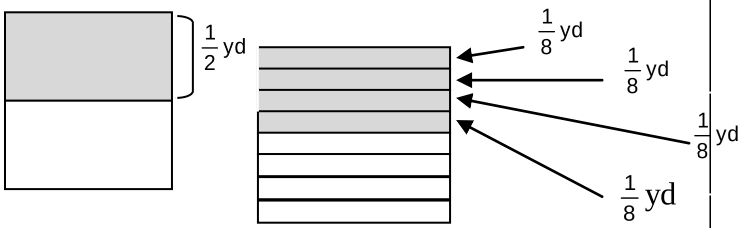
Common Misconceptions

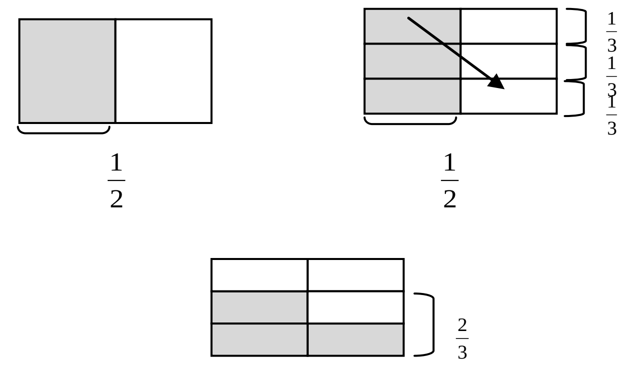
Students may believe that dividing by ½ is the same as dividing in half. Dividing by half means to find how many one-halves there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by ½ = 14 and 7 divided in half equals 3½.

Connections – Critical Areas of Focus	Connections to Other Grade Levels
This cluster is connected to part of the [Critical Area of Focus](#) for 6th grade in this domain. Through learning in **The Number System** students • Use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense.	This cluster continues the work in elementary grades from Number and Operations in Base Ten and Number and Operations – Fractions. In 7th grade, this cluster will be extended in The Number System to rational numbers and in Ratios and Proportional Reasoning.

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a*	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.	quotients fractions models equations	interpret compute solve use represent	number and operations in base 10 and fractions	In 5th grade students divided whole numbers by unit fractions. Students continue this understanding by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.  Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem: Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make? This problem can be modeled using a number line.

cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? **DOK 3**					1. Start with a number line divided into thirds.  2. The problem wants to know how many sixths are in two-thirds Divide each third in half to create sixths.  3. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards. Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer generated models. **Examples:** • 3 people share $\frac{1}{2}$ pound of chocolate. How much of a pound of chocolate does each person get? Solution: Each person gets $\frac{1}{6}$ lb of chocolate. 

					• Manny has $\frac{1}{2}$ yard of fabric to make book covers. Each book is made from $\frac{1}{8}$ yard of fabric. How many book covers can Manny make? Solution: Manny can make 4 book covers.  • Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution. **Context:** You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt from a snack pack. How much of the recipe can you make? **Explanation of Model:** The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show $\frac{1}{2}$ cup. The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally. The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model. $\frac{2}{3}$ is the new referent unit (whole) . 3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make ¾ of the recipe.

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The Number System — 6.NS

CCGPS Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.

Instructional Strategies

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in 4^{th} grade. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: "If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?" Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, $(36 + 24) = 12(3+2)$, where 12 is the GCF of 36 and 24. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in 7^{th} grade.

Instructional Resources/Tools

- **Greatest Common Factor Lesson**
  http://www.math.com/school/subject1/lessons/S1U3L2GL.html
  This lesson is a resource for teachers or for students after participating in lessons exploring GCF.

Common Misconceptions

Connections – Critical Areas of Focus

This cluster is connected to part of the Critical Area of Focus for 6^{th} grade.

Through learning in **The Number System** students

- use operations with rational numbers to solve problems.

Connections to Other Grade Levels

This cluster connects to the other 6^{th} grade clusters within The Number System Domain. It marks the final opportunity for students to demonstrate fluency with the four operations with whole numbers and decimals. In 7^{th} grade, students will extend these learnings in The Number System and in Expressions and Equations.

Standards	Mathematical Practices	What students should know	What students should do	Prerequisites	Examples/Explanations
6.NS.2. Fluently divide multi-digit numbers using the standard algorithm. **DOK 1**	**6.MP.2.** Reason abstractly and quantitatively. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.	multi-digit numbers standard algorithm	fluently divide	multiplication facts	Procedural fluency is defined by the Common Core as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately”. In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm. This understanding is foundational for work with fractions and decimals in 7th grade. Divisors can be any number of digits at this grade level. As students divide they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students’ language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, “there are 200 thirty-twos in 8456 ” and could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	There are 60 thirty twos in 2056.

					264 32)8456 −6400 2056 −1920 136 −128 There are 4 thirty twos in 136. 4 times 32 is 128. 264 32)8456 −6400 2056 −1920 136 −128 8 The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8
6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.	multi-digit decimals standard algorithm operations	fluently add fluently subtract fluently multiply fluently divide	4 basic operations with whole numbers	Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals was introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of ten to develop fluency with operations with multi-digit decimals. In 6th grade, students become fluent in the use of the standard algorithm of each of these operations. The use of estimation strategies supports student understanding of operating on decimals.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to	**6.MP.7**. Look for and make use of structure.	greatest common factor GCF whole numbers less than or equal to	find use	multiplication facts distributive property	Students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by 1. listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common

100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4*(*9+2*). **DOK 1**		least common multiple LCM distributive property sum common factor multiple			factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. IF students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor. 2. listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8). Students also understand that the greatest common factor of two prime numbers will be 1. Students use the greatest common factor and the distributive property to find the sum of two whole numbers. For example, 36 + 8 can be expressed as 4(9 + 2) = 4(11). Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by 1. listing the multiples of 6 (6, 12, 18, 24, 30, …) and 8 (8, 16, 24, 32, 40, …), then taking the least in common from the list (24); or 2. using the prime factorization. Step 1: find the prime factors of 6 and 8. 6 = 2 • 3 and 8 = 2 • 2 • 2 Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2. Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors).

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The Number System 6.NS

CCGPS Cluster: Apply and extend previous understands of numbers to the system of rational numbers.

Instructional Strategies

The purpose of this cluster is to begin study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Starting with examples of having/owing and above/below zero sets the stage for understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). They are now ready to understand that all numbers have an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple problems.

Demonstration of understanding of positives and negatives involves translating among words, numbers and models: given the words "7 degrees below zero," showing it on a thermometer and writing -7; given -4 on a number line, writing a real-life example and mathematically -4. Number lines also give the opportunity to model absolute value as the distance from zero.

Simple comparisons can be made and order determined. Order can also be established and written mathematically: -3° C > -5° C or -5° C < -3° C. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.

Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Differences between numbers can be found by counting the distance between numbers on the grid.

Instructional Resources/Tools

Common Misconceptions

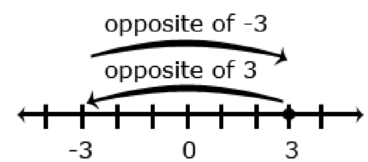
Connections – Critical Areas of Focus

This cluster does not directly address one of the Grade 6 Critical Areas of Focus. However, it is the foundation for working with rational numbers, algebraic expressions and equations, functions, and the coordinate plane in subsequent grades.

Connections to Other Grade Levels

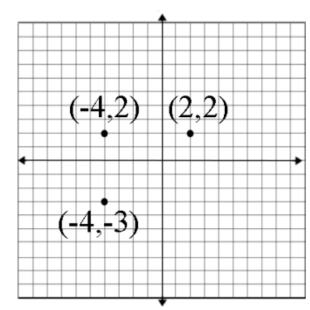
Actual computation with negatives and positives is handled in 7th grade

Standards	*Mathematical Practices*	*What students should know*	*What students should do*	*Prerequisites*	*Examples/Explanations*
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model	positive numbers negative numbers quantities opposite directions opposite values temperature above below zero elevation	understand use represent explain	number line	Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understanding the meaning of 0 in each situation. For example, 25 feet below sea level can be represented as -25; 25 feet above sea level can be represented as +25. In this scenario, zero would represent sea level.

sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **DOK 2**	with mathematics.	sea level credit debit positive electric charge negative electric charge real-world contexts meaning situation			
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.	opposite signs numbers locations opposite sides zero number line	recognize	read a number line	In elementary school, students worked with positive fractions, decimals and whole numbers on the number line. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer). Students recognize that a number and its opposite are equidistant from zero (reflections about the zero). The opposite sign (-) shifts the number to the opposite side of 0. For example, -4 could be read as "the opposite of 4" which would be negative 4. The following example, -(06.4) would be read as "the opposite of the opposite 6.4" which would be 6.4. Zero is its own opposite. Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids. 

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.	signs ordered pairs locations quadrants coordinate plane differ points reflections axes	understand recognize	coordinate system plotting points reflections	Students worked with Quadrant I in elementary school. As the x-axis and y-axis are extending to include negatives, students begin to work with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersects as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (-, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change in the x-coordinates from (-2, 4) to (2, 4), represents a reflection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.	integers rational numbers horizontal vertical number line diagram pairs coordinate plane	find position	plot numbers on a number line plot points on a coordinate plane	Students are able to plot all rational numbers on a number line (either vertical or horizontal) or identify the values of given points on a number line. For example, students are able to identify where the following numbers would be on a number line: -4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, -2, $\frac{11}{2}$.
6.NS.7. Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –*	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason	statements of inequality relative position numbers number line	interpret	order rational numbers on a number line compare rational numbers on a number line	Students identify the absolute value of a number as the distance from zero but understand that although the value of -7 is less than -3, the absolute value (distance) of -7 is greater than the absolute value (distance) of -3. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. For example, –4 ½ < –2 because –4 ½ is located to the left of –2 on the number line.

3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. **DOK 1**	abstractly and quantitatively. **6.MP.4**. Model with mathematics.				
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.	statements of order rational numbers real-world contexts	write interpret explain	order rational numbers on a number line compare rational numbers on a number line	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than". For example, the balance in Sue's checkbook was –12.55. The balance in John's checkbook was –10.45. Since –12.55 < –10.45, Sue owes more than John. The interpretation could also be "John owes less than Sue".
c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* **DOK 1**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.	absolute value rational number distance from 0 magnitude positive quantity negative quantity real-world situation	understand interpret	definition of absolute value opposites on a number line	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. For example, | –7 | can be interpreted as the distance –7 is from 0 which would be 7. Likewise | 7 | can be interpreted as the distance 7 is from 0 which would also be 7. In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of –900 feet, write | –900| = 900 to describe the distance below sea level.
d. Distinguish comparisons of absolute value from statements about order. *For example,*	**6.MP.1**. Make sense of problems and persevere in solving them.	absolute value statements of order	distinguish	definition of absolute value compare and order rational numbers	When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the

recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics.				negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than –14. For negative numbers, as the absolute value increases, the value of the number decreases.
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.7**. Look for and make use of structure.	real-world problems mathematical problems points quadrants coordinate plane coordinates absolute value distance	solve graph find	graph points in all 4 quadrants understand absolute value	Students find the distance between points whose ordered pairs have the same *x*-coordinate (vertical) or same *y*-coordinate (horizontal). For example, the distance between (–5, 2) and (–9, 2) would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between 9 and 5. (| 9 | – | 5 |). Coordinates could also be in two quadrants. For example, the distance between (3, –5) and (3, 7) would be 12 units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from –5 to 7 or by recognizing that the distance (absolute value) from –5 to 0 is 5 units and the distance (absolute value) from 0 to 7 is 7 units so the total distance would be 5 + 7 or 12 units. Example: • If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle?  [Return to Contents](#)

Expressions and Equations 6.EE

CCGPS Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Strategies

The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar $\left(\frac{3}{5}\right)$. Less confusion will occur as students write algebraic expressions and equations if x represents only variables and not multiplication. The use of a dot (·) or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees,"', "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \cdot 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in the 6th grade standards in The Number System domain; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

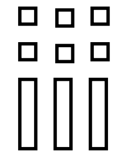
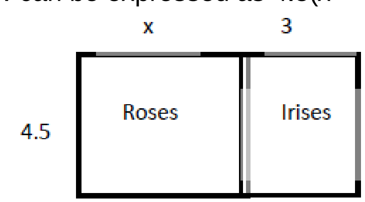
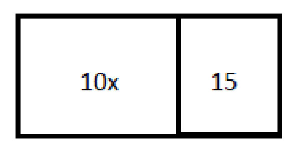
Instructional Resources/Tools	Common Misconceptions
• **Algebra Tiles** (from the National Library of Virtual Manipulatives) http://nlvm.usu.edu/en/nav/frames_asid_189_g_3_t_2.html?open=activities&from=category_g_3_t_2.html Online algebra tiles can be used to represent expressions and equations. • **Late Delivery** http://www.bbc.co.uk/education/mathsfile/shockwave/games/postie.html In this game, the student helps the mail carrier deliver five letters to houses with numbers such as $3(a + 2)$.	Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \cdot x \cdot x$ which is x times x times x, not $3x$ or 3 times x; $4x$ means 4 times x or x+x+x+x, not forty-something. When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47. Use of the "x" notation as both the variable and the operation of multiplication can complicate this understanding.

Connections – Critical Areas of Focus	Connections to Other Grade Levels
This cluster is connected to the 6th grade Critical Area of Focus, **Writing, interpreting and using expressions, and equations**.	The learning in this cluster is foundational in the transition to algebraic representation and problem solving which is extended and formalized in 7th grade, the Number System and Expressions and Equations.

Standards	*Mathematical Practices*	*What students should know*	*What students should do*	*Prerequisites*	*Examples/Explanations*
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively.	numerical expressions whole numbers exponents	write evaluate	write expressions interpret expressions	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e., $\left(\frac{1}{2}\right)^5$ can be written $\frac{1}{2}\bullet\frac{1}{2}\bullet\frac{1}{2}\bullet\frac{1}{2}\bullet\frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (i.e., x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. Examples: • Write the following as a numerical expressions using exponential notation. ○ The area of a square with a side length of 8 m (Solution: $8^2 m^2$) ○ The volume of a cube with a side length of 5 ft: (Solution: $5^3 ft^3$) ○ Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own: (Solution: 2^3 mice) • Evaluate: ○ 4^3 (Solution: 64) ○ $5+2^4\bullet 6$ (Solution: 101) ○ $7^2 - 24 \div 3 + 26$ (Solution: 67)
6.EE.2. Write, read, and evaluate expressions in which letters stand for	**6.MP.1**. Make sense of problems and persevere in	expressions operations	write	4 basic operations	Students write expressions from verbal descriptions using letters and numbers. Students understand order is

numbers. **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* **DOK 1**	solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.6**. Attend to precision.	numbers letters		constants variables	important in writing subtraction and division problems and that a number and letter written together means to multiply. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. • r + 21 as "some number plus 21 as well as "r plus 21" • n • 6 as "some number times 6 as well as "n times 6" • $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6"
b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8+7) as a product of two factors; view (8+7) as both a single entity and a sum of two terms*. **DOK 1**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.6**. Attend to precision.	parts expression mathematical terms sum term product factor quotient coefficient parts of an expression single entity	identify view	multiplication and addition properties	Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes. Consider the following expression: $x^2+5y+3x+6$ The variables are x and y. There are 4 terms: x^2, 5y, 3x, and 6. There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of

					x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5 y's or 5 • y. There is one constant term, 6. The expression shows a sum of all four terms. Students can also describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V=s^3$ and $A=6s^2$ to find the volume and surface area of a cube with sides of length $s=1/2$.* **DOK 1**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.6**. Attend to precision.	expressions values variables formulas real-world problems arithmetic operations whole numbers exponents order parentheses order of operations	evaluate perform	order of operations simplify expressions with exponents	Students evaluate algebraic expressions, using order of operations as needed. Given an expression such as 3*x* + 2 *y*, find the value of the expression when *x* is equal to 4 and *y* is equal to 2.4. This problem requires students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. 3 • 4 + 2 • 2.4 12 + 4.8 16.8 Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. For example, it costs $100 to rent the skating rink plus $5 per person. The cost for any number (*n*) of people could be found by the expression, 100 + 5*n*. What is the cost for 25 people?
6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to*	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable	properties of operations equivalent expressions	apply generate	mathematical properties of operations	Students use their understanding of multiplication to interpret *3 (2 + x). For example, 3 groups of (2 + x).* They use a model to represent x, and make an array to show the meaning of *3(2 + x).* They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x.*

the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. **DOK 2**	arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure.				An array with 3 columns and *x* + 2 in each column:  Students interpret *y* as referring to one *y.* Thus, they can reason that one *y* plus one *y* plus one *y* ***must be*** *3y.* They also the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$: $y + y + y = y \times 1 + y \times 1 + y \times 1 = y \times (1 + 1 + 1) = y \times 3 = 3y$ Students use the distributive property to write equivalent expressions. For example, area models from elementary can be used to illustrate the distributive property with variables. Given that the width is 4.5 units and the length can be represented by *x* + 3, the area of the flowers below can be expressed as 4.5(*x* + 3) or 4.5*x* + 13.5.  When given an expression representing area, students need to find the factors. For example, the expression 10*x* + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2*x* + 3). The factors (dimensions) of this figure would be 5(2*x* + 3). 

<table>
<tr>
<td>6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.
DOK 1</td>
<td>6.MP.2. Reason abstractly and quantitatively.

6.MP.3. Construct viable arguments and critique the reasoning of others.

6.MP.4. Model with mathematics.

6.MP.6. Attend to precision.

6.MP.7. Look for and make use of structure.</td>
<td>expressions
equivalent
number value</td>
<td>identify</td>
<td>mathematical operations</td>
<td>Students connect their experiences with finding and identifying equivalent forms of whole numbers and can write expressions in various forms. Students generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example:
Are the expressions equivalent? How do you know?
4m + 8
4(m+2)
3m + 8 + m
2 + 2m + m + 6 + m
Solution:
<table>
<tr><td>Expression</td><td>Simplifying the Expression</td><td>Explanation</td></tr>
<tr><td>4m + 8</td><td>4m + 8</td><td>Already in simplest form</td></tr>
<tr><td>4(m+2)</td><td>4(m+2)
4m + 8</td><td>Distributive property</td></tr>
<tr><td>3m + 8 + m</td><td>3m + 8 + m
3m + m + 8
(3m + m) + 8
4m + 8</td><td>Combined like terms</td></tr>
<tr><td>2 + 2m + m + 6 + m</td><td>2 + 2m + m + 6 + m
2 + 6 + 2m + m + m
(2 + 6) + (2m + m + m)
8 + 4m
4m + 8</td><td>Combined like terms</td></tr>
</table></td>
</tr>
</table>

Expressions and Equations — 6.EE

CCGPS Cluster: Reason about and solve one-variable equations and inequalities.

Instructional Strategies

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; Students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

Instructional Resources/Tools

Common Misconceptions

Connections – Critical Areas of Focus

This cluster is connected to the Critical Area of Focus for 6th grade, **Writing, interpreting and using expressions, and equations.**

Connections to Other Grade Levels

Standards	*Mathematical Practices*	*What students should know*	*What students should do*	*Prerequisites*	*Examples/Explanations*
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively.	equation inequality process question values set true substitution number	understand use determine	substitution comparing and ordering numbers	Students identify values from a specified set that will make an equation true. For example, given the expression $x + 2\frac{1}{2}$, which of the following values for x would make $x + 2\frac{1}{2} = 6$? {0, $3\frac{1}{2}$, 4} By using substitution, students identify 3 ½ as the value that will make both sides of the equation equal. The solving of inequalities is limited to choosing values from a specified set that would make the inequality true.

inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **DOK 1**	**6.MP.4**. Model with mathematics. **6.MP.7**. Look for and make use of structure.				For example, find the value(s) of x that will make x + 3.5 ≥ 9. {5, 5.5, 6, $\frac{15}{2}$, 10.2, 15} Using substitution, students identify 5.5, 6, $\frac{15}{2}$, 10.2, and 15 as the values that make the inequality true. NOTE: If the inequality had been x + 3.5 > 9, then 5.5 would not work since 9 is not greater than 9. This standard is foundational to **CC.6.EE.7** and **CC.6.EE.8**.
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **DOK 2**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.7**. Look for and make use of structure.	variable number expressions real-world problem mathematical problem set	use represent write solve understand	variables mathematical expressions	Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. **Examples:** • Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. (Solution: 2*c* + 3 where *c* represents the number of crayons that Elizabeth has.) • An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: 28 + 0.35*t* where *t* represents the number of tickets purchased) • Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned. (Solution: 15*h* + 20 = 85 where *h* is the number of hours worked) • Describe a problem situation that can be solved using the equation 2*c* + 3 = 15; where *c* represents the cost of an item • Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. (Solution: $5.00 + *n*)
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason	real-world problems mathematical problems equations nonnegative rational numbers	solve write	understand inverse operations	Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, *x* + 4, any value can be substituted for the *x* to generate a numerical answer; however, in the equation *x* + 4 = 6, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

<table>
<tr>
<td>which p, q and x are all nonnegative rational numbers
DOK 2</td>
<td>abstractly and quantitatively.

6.MP.3. Construct viable arguments and critique the reasoning of others.

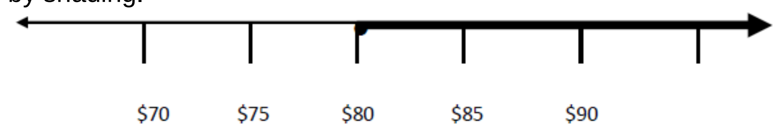
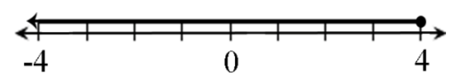
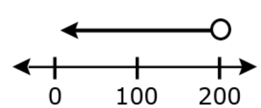
6.MP.4. Model with mathematics.

6.MP.7. Look for and make use of structure.</td>
<td></td>
<td></td>
<td></td>
<td>Students write equations from real-world problems and then use inverse operations to solve one-step equations. Equations may include fractions and decimals with non-negative solutions.

Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Examples:

Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.
<table>
<tr><td colspan="3">$56.58</td></tr>
<tr><td>J</td><td>J</td><td>J</td></tr>
</table>
Sample Solution: Students might say: “I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation 3J = $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That’s $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15+3+0.86). I double check that the jeans cost $18.86 each because $18.86 x 3 is $56.58.”
Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.
(Solution: 20 = 1.99 + 6.50 + x, x = $11.51)
<table>
<tr><td colspan="3">20</td></tr>
<tr><td>1.99</td><td>6.50</td><td>money left over (m)</td></tr>
</table>
</td>
</tr>
</table>

6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **DOK 1**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.7**. Look for and make use of structure.	inequality constraint condition real-world problem mathematical problem infinitely many solutions number line diagrams	write represent recognize	comparing and ordering numbers on a number line	Many real-world situations are represented by inequalities. Students write an inequality and represent solutions on a number line for various contextual situations. For example, the class must raise at least $80 to go on the field trip. If *m* represents money, then the relationship can be expressed using the inequality $m \geq$ to $80. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.  A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions. **Examples:** • Graph $x \leq 4$.  • Jonas spent more than $50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line. • Less than $200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line. Solution*:* $200 > x$  *Return to Contents*

Expressions and Equations 6.EE

CCGPS Cluster: Represent and analyze quantitative relationships between dependent and independent variables.

Instructional Strategies

The goal is to help students connect the pieces together. This can be done by having students use multiple representations for the mathematical relationship. Students need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs and equations. They also need to be able to start with any of the representations and develop the others.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate that I am traveling.

Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

The use of technology, including computer apps, CBLs, and other hand-held technology allows the collection of real-time data or the use of actual data to create tables and charts. It is valuable for students to realize that although real-world data often is not linear, a line sometimes can model the data.

Instructional Resources/Tools

- Use graphic organizers as tools for connecting various representations.
- **Pedal Power**
  http://illuminations.nctm.org/LessonDetail.aspx?id=L586
  NCTM illuminations lesson on translating a graph to a story.

Common Misconceptions

Students may misunderstand what the graph represents in context. For example, that moving up or down on a graph does not necessarily mean that a person is moving up or down.

Connections – Critical Areas of Focus

This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations**.

Connections to Other Grade Levels

This cluster, Expressions and Equations, is closely tied to Ratios and Proportional Relationships, allowing the ideas in each to be connected and taught together.

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable	variables quantities real-world problem relationship equation dependent variable independent variable graphs tables	use represent write express analyze relate	using graphs and tables	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the *x*-axis; the dependent variable is graphed on the *y*-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be

variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **DOK 2**	arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.				represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the *x* variable increases, how does the *y* variable change?) Relationships should be proportional with the line passing through the origin. Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values. Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.

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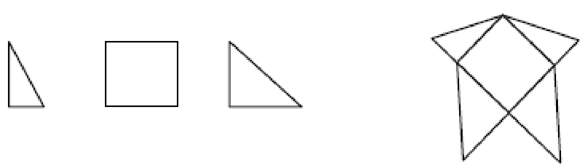
Geometry 6.G

CCGPS Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.

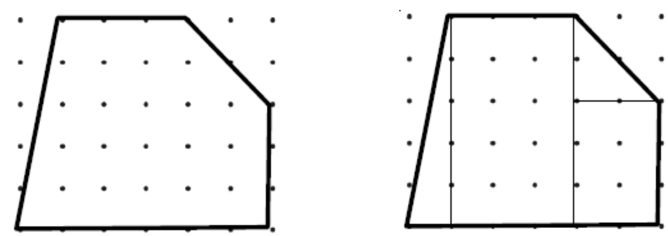
Instructional Strategies

It is very important for students to continue to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. Exploring possible nets should be done by taking apart (unfolding) three-dimensional objects. This process is also foundational for the study of surface area of prisms. Building upon the understanding that a net is the two-dimensional representation of the object, students can apply the concept of area to find surface area. The surface area of a prism is the sum of the areas for each face.

Multiple strategies can be used to aid in the skill of determining the area of simple two-dimensional composite shapes. A beginning strategy should be to use rectangles and triangles, building upon shapes for which they can already determine area to create composite shapes. This process will reinforce the concept that composite shapes are created by joining together other shapes, and that the total area of the two-dimensional composite shape is the sum of the areas of all the parts.



A follow-up strategy is to place a composite shape on grid or dot paper. This aids in the decomposition of a shape into its foundational parts. Once the composite shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.



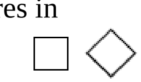
Fill prisms with cubes of different edge lengths (including fractional lengths) to explore the relationship between the length of the repeated measure and the number of units needed. An essential understanding to this strategy is the volume of a rectangular prism does not change when the units used to measure the volume changes. Since focus in Grade 6 is to use fractional lengths to measure, if the same object is measured using one centimeter cubes and then measured using half centimeter cubes, the volume will appear to be eight times greater with the smaller unit. However, students need to understand that the value or the number of cubes is greater but the volume is the same.

Instructional Resources/Tools

- Cubes of fractional edge length
- Squares that can be joined together used to develop possible nets for a cube
- Use floor plans as a real world situation for finding the area of composite shapes.
- **Online dot paper**
  http://illuminations.nctm.org/lessons/DotPaper.pdf#search=%22dot paper%22
- **Lessons on area**
  http://illuminations.nctm.org/LessonDetail.aspx?ID=L580

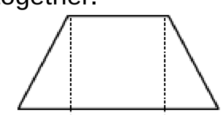
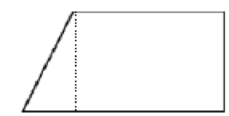
Common Misconceptions

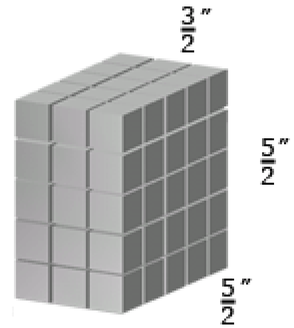
Students may believe that the orientation of a figure changes the figure. In Grade 6, some students still struggle with recognizing common figures in different orientations. For example, a square rotated 45° is no longer seen as a square and instead is called a diamond.



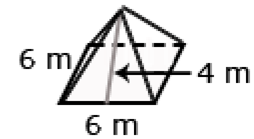
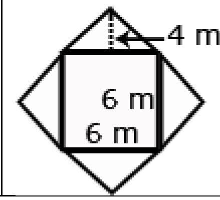
This impacts students' ability to decompose composite figures and to appropriately apply formulas for area. Providing multiple orientations of objects within classroom examples and work is essential for students to overcome this misconception.

Connections – Critical Areas of Focus	Connections to Other Grade Levels
This cluster does not directly relate to one of the Critical Areas of Focus for 6th grade. This cluster focuses on additional content for development. Students in 6th grade build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume.	An understanding of how to find the area, surface area and volume of an object is developed in 5th grade and should be built upon in 6th grade to facilitate understanding of the formulas found in Measurement and Data and when to use the appropriate formula. The use of floor plans and composite shapes on dot paper is a foundational concept for scale drawing and determining the actual area based on a scale drawing 7th grade (Geometry and Ratio and Proportional Relationships).

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure.	area right triangles triangles quadrilaterals polygons rectangles shapes real-world problems mathematical problems	find compose decompose apply	how to find area of triangles and rectangles	Students continue to understand that area is the number of squares needed to cover a plane figure. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ *bh* or ($b \times h$)/2. Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figure below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together.  Isosceles trapezoid  Right trapezoid Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on NCTM's Illuminations site to shift, rotate, color, decompose and view figures in 2D or 3D (http://illuminations.nctm.org/ActivityDetail.aspx?ID=125)

	6.MP.8. Look for and express regularity in repeated reasoning.				
6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas *V* = *l w h* and *V* = *b h* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.	volume right rectangular prism fractional edge length unit cubes unit fraction formula real-world problems mathematical problems	find pack show multiply apply solve	length width height multiplication use of formulas	Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The unit cube was $1 \times 1 \times 1$. In 6th grade the unit cube will have fractional edge lengths (i.e., ½ • ½ • ½). Students find the volume of the right rectangular prism with these unit cubes. For example, the model below shows a rectangular prism with dimensions 3/2 inches, 5/2 inches, and 5/2 inches. Each of the cubic units in the model is ½ in^3. Students work with the model to illustrate 3/2 x 5/2 x 5/2 = (3 x 5 x 5) x 1/8. Students reason that a small cube has volume 1/8 because 8 of them fit in a unit cube or ½ x ½ x ½ = 1/8.  “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of ***why*** the formula works and how the formula relates to the measure (volume) and the figure. This understanding should be for *all* students. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM’s Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or **th**e same second coordinate. Apply **t**hese techniques in the context of solving real-world and mathematical problems. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.7**. Look for and make use of structure.	polygons coordinate plane coordinates vertices length side points real-world problems mathematical problems	draw use find apply solve	plotting points in the coordinate plane addition subtraction	Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same [for example, (2, –1) and (2, 4)], then students recognize that a vertical line has been created and the distance between these coordinates is the distance between –1 and 4, or 5. If both the *y*-coordinates are the same [e.g., (–5, 4) and (2, 4)], then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between –5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area of quadrilaterals and triangles. This standard can be taught in conjunction with **CC.6.G.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the area of the triangle is ½ the area of the associated square, rectangle, or parallelogram. Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle.
6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics.	three-dimensional figures nets rectangles triangles surface area real-world problems mathematical problems	represent use find apply solve	recognize 2D and 3D figures calculate surface area	A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205). Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure. **Examples:** • Describe the shapes of the faces needed to construct

	6.MP.5. Use appropriate tools strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure. **6.MP.8**. Look for and express regularity in repeated reasoning.				a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? • Create the net for a given prism or pyramid, and then use the net to calculate the surface area.  

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Statistics and Probability 6.SP

CCGPS Cluster: Develop understanding of statistical variability.

Instructional Strategies

Grade 6 is the introduction to the formal study of statistics for students. Students need multiple opportunities to look at data to determine and word statistical questions. Data should be analyzed from many sources, such as organized lists, box-plots, bar graphs and stem-and-leaf plots. This will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. At the same time, students should begin to relate their informal knowledge of mean, mode and median to understand that data can also be described by single numbers. The single value for each of the measures of center (mean, median or mode) and measures of spread (range, interquartile range, mean absolute deviation) is used to summarize the data. Given measures of center for a set of data, students should use the value to describe the data in words. The important purpose of the number is not the value itself, but the interpretation it provides for the variation of the data. Interpreting different measures of center for the same data develops the understanding of how each measure sheds a different light on the data. The use of a similarity and difference matrix to compare mean, median, mode and range may facilitate understanding the distinctions of purpose between and among the measures of center and spread.

Include activities that require students to match graphs and explanations, or measures of center and explanations prior to interpreting graphs based upon the computation measures of center or spread. The determination of the measures of center and the process for developing graphical representation is the focus of the cluster "Summarize and describe distributions" in the Statistics and Probability domain for Grade 6. Classroom instruction should integrate the two clusters.

Instructional Resources/Tools

- Newspaper and magazine graphs for analysis of the spread, shape and variation of data
- **Numerical and Categorical Data**
  http://illuminations.nctm.org/LessonDetail.aspx?ID=L368
  In this unit of three lessons, students formulate and refine questions, and collect, display and analyze data.
- **Data Analysis and Probability Virtual Manipulatives Grades 6-8**
  http://nlvm.usu.edu/en/nav/category_g_3_t_5.html
  Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without requiring students to spend time hand-drawing the display. Classroom time can then be spent discussing the patterns and variability of the data.
- Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistics Association
  http://www.amstat.org/education/gaise/GAISEPreK-12_Full.pdf

Common Misconceptions

Students may believe all graphical displays are symmetrical. Exposing students to graphs of various shapes will show this to be false.

The value of a measure of center describes the data, rather than a value used to interpret and describe the data.

Connections – Critical Areas of Focus

This cluster is connected to the Critical Area of Focus for 6th grade, **Developing Understanding of statistical thinking**.

Connections to Other Grade Levels

Measures of center and measures of variability are used to draw informal comparative inferences about two populations in CC.7.SP.4.

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **DOK 2**	**6.MP.1**. Make sense of problems and persevere in solving them. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.6**. Attend to precision.	statistical question variability data answer	recognize	statistical thinking	Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Questions can result in a narrow or wide range of numerical values. For example, asking classmates "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?" Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?" To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.
6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools	set data statistical question distribution center spread shape	understand	measures of center and spread	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots.

	strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure.				
6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **DOK 1**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure.	measure of center numerical data set values number measure of variation vary	recognize	measures of center and variation	Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (i.e., midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variation are used to describe this characteristic.

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CCGPS Cluster: Summarize and describe distributions.

Instructional Strategies

This cluster builds on the understandings developed in the 6th grade cluster "Develop understanding of statistical variability." Students have analyzed data displayed in various ways to see how data can be described in terms of variability. Additionally, in grades 3 – 5 students have created scaled picture and bar graphs, as well as line plots. Now students learn to organize data in appropriate representations such as box plots (box-and-whisker plots), dot plots, and stem-and-leaf plots. Students need to display the same data using different representations. By comparing the different graphs of the same data, students develop an understanding of the benefits of each type of representation.

Further interpretation of the variability comes from the range and center-of-measure numbers. Prior to learning the computation procedures for finding mean and median, students will benefit from concrete experiences.

To find the median visually and kinesthetically, students should reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median.

The concept of mean (concept of fair shares) can be demonstrated visually and kinesthetically by using stacks of linking cubes. The blocks are redistributed among the towers so that all towers have the same number of blocks. Students should not only determine the range and centers of measure, but also use these numbers to describe the variation of the data collected from the statistical question asked. The data should be described in terms of its shape, center, spread (range) and interquartile range or mean absolute deviation (the absolute value of each data point from the mean of the data set). Providing activities that require students to sketch a representation based upon given measures of center and spread and a context will help create connections between the measures and real-life situations.

Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of spread and measures of center mathematically need to follow the development of the conceptual understanding. Students should experience data which reveals both different and identical values for each of the measures. Students need opportunities to explore how changing a part of the data may change the measures of center and measure of spread. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability, what each of the measures do and do not tell about a set of data, all leading to a better understanding of their usage.

Using graphing calculators to explore bos plots (box-and-whisker plots) removes the time intensity from their creation and permits more time to be spent on the meaning. It is important to use the interquartile range in box plots when describing the variation of the data. The mean absolute deviation describes the distance each point is from the mean of that data set. Patterns in the graphical displays should be observed, as should any outliers in the data set. Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data. Pairing contextual situations with data and its box-and-whisker plot is essential.

Instructional Resources/Tools

- Graphing calculators may be used for creating lists and displaying the data.
- Guidelines for Assessment and Instruction in Statistics Education (GAISE) report, American Statistical Association
  http://www.amstat.org/education/gaise/GAISEPreK-12_Full.pdf
- **Hollywood Box Office**
  http://www.ohiorc.org/pm/math/richproblemmath.aspx?pmrid=62
  This rich problem focuses on measures of center and graphical displays.

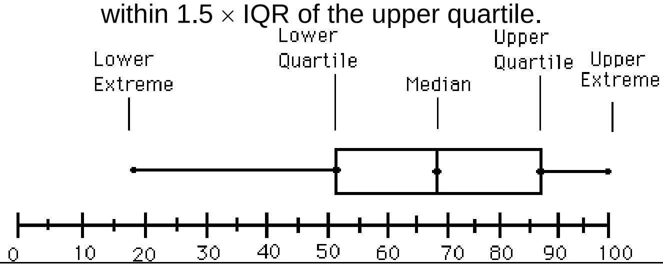
(Continued on next page)

Common Misconceptions

Students often use words to help them recall how to determine the measures of center. However, student's lack of understanding of what the measures of center actually represent tends to confuse them. Median is the number in the middle, but that middle number can only be determined after the data entries are arranged in ascending or descending order. Mode is remembered as the "most," and often students think this means the largest value, not the "most frequent" entry in the set. Vocabulary is important in mathematics, but conceptual understanding is equally as important. Usually the mean, mode, or median have different values, but sometimes those values are the same.

• **Wet Heads** http://www.pbs.org/teachers/mathline/lessonplans/msmp/wetheads/wetheads_procedure.shtm In this lesson, students create stem-and-leaf plots and back-to-back stem-and-leaf plots to display data collected from an investigative activity. • **Stella's Stumpers Basketball Team Weight** http://ohiorc.org/for/math/stella/problems/problem.aspx?id=438 This problem situation uses the mean to determine a missing data element. • **Learning Conductor Lessons.** http://ohiorc.org/for/math/learningconductor/lessons.aspx Use the interactive applets in these standards-based lessons to improve understanding of mathematical concepts. Scroll down to the **statistics** section for your specific need. • **Height of Students in Our Class** http://illuminations.nctm.org/LessonDetail.aspx?ID=L231 This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot. • **National Library of Virtual Manipulatives** http://nlvm.usu.edu/en/nav/category_g_3_t_5.html Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without the tediousness of the student hand drawing the display.	The use of a similarities and differences matrix to compare mean, median, mode, range, interquartile range, and mean absolute deviation will facilitate student understanding of the uniqueness of these values. **Connections – Critical Areas of Focus** This cluster is connected to the Critical Area of Focus for 6th grade, **Developing understanding of statistical thinking**. **Connections to Other Grade Levels** Measures of center and measures of variability are used to draw informal comparative inferences about two populations in 7th grade Statistics and Probability.

Standards	***Mathematical Practices***	***What students should know***	***What students should do***	***Prerequisites***	***Examples/Explanations***
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **DOK 2**	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically.	numerical data plots number line dot plots histograms box plots	display	number lines and graphing	Students display data set using number lines. Dot plots, histograms and box plots are three graphs to be used. A dot plot is a graph that uses a point (dot) for each piece of data. The plot can be used with data sets that include fractions and decimals. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers. A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many

<table>
<tr><td></td><td> 6.MP.6. Attend to precision. 6.MP.7. Look for and make use of structure. </td><td></td><td></td><td></td><td> numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it. Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. A box plot shows the distribution of values in a data set by dividing the set into quartiles. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data. These values give a summary of the shape of a distribution. Students understand that the box and whiskers represent the quartiles in the distribution of the data. For horizontal box and whisker plots, the left side and right sides of the box are always the 25th and 75th percentiles (the lower and upper quartiles, respectively), and the band inside the box is always the 50th percentile (the median). [For vertical box and whisker plots, the bottom and the top of the box are always the 25th and 75th percentiles. As with the horizontal plots, the band inside the box is always the 50th percentile.] The ends of the whiskers can represent several possible alternative values, such as the minimum and maximum of all the data
the lowest value in the set which is within 1.5 × IQR of the lower quartile, and the highest data value in the set within 1.5 × IQR of the upper quartile.

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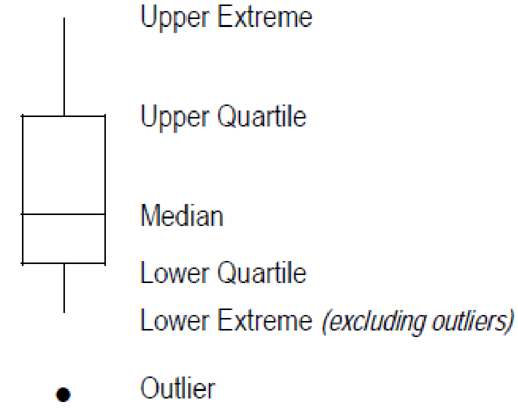
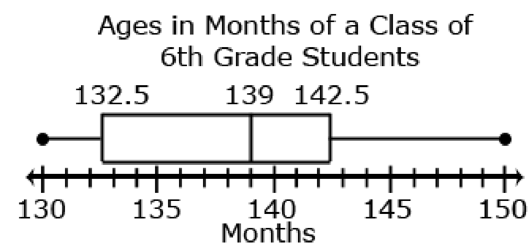


Example:

- Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

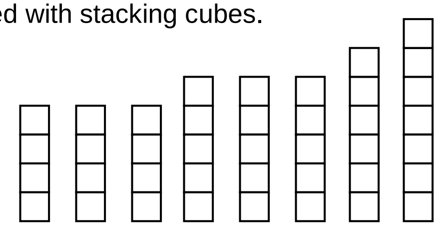
Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months



					This box plot shows that • ¼ of the students in the class are from 130 to 132.5 months old • ¼ of the students in the class are from 142.5 months to 150 months old • ½ of the class are from 132.5 to 142.5 months old • the median class age is 139 months. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations. Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77 Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78
6.SP.5. Summarize numerical data sets in relation to their context, such as by: **a.** Reporting the number of observations. **DOK 1** **b.** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **DOK 1** **c.** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any	**6.MP.2**. Reason abstractly and quantitatively. **6.MP.3**. Construct viable arguments and critique the reasoning of others. **6.MP.4**. Model with mathematics. **6.MP.5**. Use appropriate tools strategically. **6.MP.6**. Attend to precision. **6.MP.7**. Look for and make use of structure.	number observations nature attribute investigation units of measurement quantitative measures of center median mean quantitative measures of variability interquartile range mean absolute deviation	report describe give describe	units of measurement measures of center and variability interquartile range finding patterns understand the context of a problem	Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). Consideration may need to be given to how the data was collected (i.e., random sampling). Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will

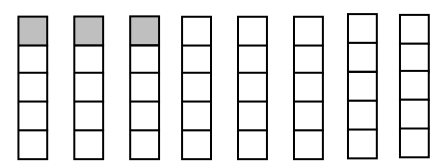
overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **DOK 3** **d.** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. **DOK 3**		pattern deviation context choice measures of center measures of variability shape data distribution context	relate	measures of center and variability understand the context of a problem	be different, with the median frequently providing a better overall description of the data set. The mean is the arithmetic average or balance point of a distribution. The mean is the sum of the values in a data set divided by how many values there are in the data set. The mean represents the value if all pieces of the data set had the same value. As a balancing point, the mean is the value where the data values above and the data values below have the same value. Measures of variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers. The Mean Absolute Deviation describes the variability of the data set by determining the absolute value deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data. Students understand how the measures of center and measures of variability are represented by the graphical display. Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. <u>Understanding the Mean</u> The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.

For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes. Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data set could be represented with stacking cubes.



Students can model the mean by "leveling" the stacks or distributing the blocks so the stacks are "fair". Students are seeking to answer the question "If all of the students had the same number of letters in their name, how many letters would each person have?"

One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.

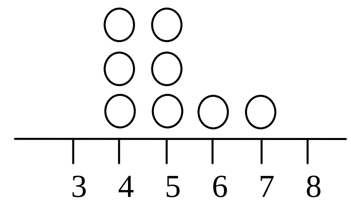


If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.

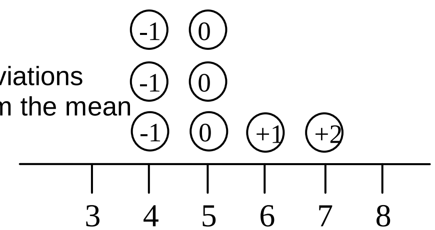
<u>Understanding Mean Absolute Deviation</u>

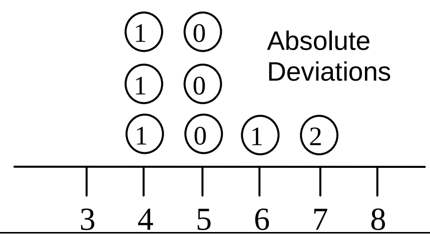
The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.



To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.





Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or ¾ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5.

$$\frac{(3+3+3+3+7+7+7+7)}{8} = \frac{40}{8} = 5$$

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Sue	3	-2	2
Joe	3	-2	2
Jim	3	-2	2
Amy	3	-2	2
Sabrina	7	+2	2
Timothy	7	+2	2
Adelita	7	+2	2
Monique	7	+2	2
Total	40	0	16

The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.

					<u>Understanding Medians and Quartiles</u> Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed. Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest. 5 4 5 4 7 6 4 5 $\longrightarrow$ 4 4 4 5 5 5 6 7 The middle value in the ordered data set is the median. If there are an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values which are both 5. Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2nd and 3rd value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data. 4 4 4 5 5 5 6 7 Q1 = 4 Q3 = 5.5 Median = 5